AQA Computer Science A-Level

4.3.3 Reverse Polish

Intermediate Notes

Specification:

4.3.3.1 Reverse Polish – infix transformations

Be able to convert simple expressions in infix form to Reverse Polish notation (RPN) form and vice versa. Be aware of why and where it is used. Eliminates need for brackets in sub-expressions. Expressions in a form suitable for evaluation using a stack. Used in interpreters based on a stack for example Postscript and bytecode.

Infix Notation

Humans prefer to use in-fix order of notation. This means that the operand is either side of the opcode. However, longer equations can cause confusion over the order of execution.

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Opcode is the **instruction** (e.g. + / MOV). **Operand** refers to the **data** on which the **operation** is being performed. In 1 + 2, one and two are the operand, and + is the opcode.

Opcode and Operands are covered in **Structure and role of the Processor and its Components** under **Fundamentals of Computer Organisation and Architecture**.

Example 1:

$$3 + 5$$

3 and 5 are the operand and + is the opcode. The answer is 8.

Example 2:

$$9 + 6 / 3$$

BODMAS

Determines the order of execution of an equation. In order of priority: Brackets, Orders (powers/indices), Division, Multiplication, Addition, Subtraction. According to this rule, the answer to 4 + 8 / 2 - 3 is 5.

These expressions can either use brackets or BODMAS to alleviate the confusion.
According to BODMAS, the following equation is produced.

$$9 + 6 / 3 = 11$$

However, brackets could be added to produce an equation with a different answer.

$$(9 + 6) / 3 = 5$$

Reverse Polish Notation

Reverse Polish Notation (RPN) is a postfix way of writing expressions. This eliminates the need for brackets and any confusion over the order of execution. Rather than the opcode going in between the operand, a postfix expression writes the opcode after the operand. When the opcode has both pieces of operand immediately preceding it, the operation proceeds.

> **Reverse Polish Notation**
>
> **Also known as postfix notation or RPN. The operators follow the operand.**

Example 1:

This is an infix equation.

$$3 + 5$$

This is its postfix equivalent.

$$3\ 5\ +$$

They both give the answer 8.

Example 2:

This is an infix equation. Its answer is 11.

$$9 + 6 / 3$$

This is its postfix equivalent.

$$9\ 6\ 3\ /\ +$$

Proof

The / sign has two pieces of operand immediately before it (6 and 3).

$$9\ 6\ 3\ /\ +$$

It performs the operation 6 / 3, which equals 2.

$$9\ 6\ 3\ /\ +$$

$$6\ 3\ / = 6 / 3 = 2$$

$$9\ 2\ +$$

Now the postfix expression reads 9 2 +. The 9 and the 2 are immediately before the plus sign.

9 2 +

They are added together to make 11, the same as its infix equivalent.

9 2 +

9 2 + = 9 + 2 = 11

11

Converting from Infix to Postfix

> **Synoptic Link**
>
> **Traversal** is the process of **visiting** each **node** in a **graph**. There are several types, each outputting the nodes in a different order. **Postorder traversal** is exclusively for **trees**.
>
> Traversals are covered in **Graph-traversal** and **Tree-traversal**, both under **Fundamentals of Data Structures**.

Infix expressions can be converted into postfix by the postorder traversal of an expression tree. Simpler ones can be done by observation.

> **Synoptic Link**
>
> **Graphs** can be used as **visual representations** of **complex relationships**. A **tree** is an **acyclic connected graph**. An **expression tree** is a **binary tree** with **operand** and **opcode** as **nodes**.
>
> Graphs are covered in **Graphs** under **Fundamentals of Data Structures**. Trees are covered in **Trees** under **Fundamentals of Data Structures.**

Example 1:

The following expression needs to be converted into its postfix equivalent.

$$((y - 6) / 3) * (x + 4)$$

> **Note**
>
> This method will work by choosing any operator first, but it is a smart idea to work from left to right as to not forget any part of the expression. Do not remove the brackets until the end.

The first operator is selected.

$$((y - 6) / 3) * (x + 4)$$

The minus sign is our first opcode. Because of the brackets around the operation, the two pieces of operand are 12 and 6. 12 - 6 is the same as 12 6 - in RPN, so this part of the equation can be replaced.

$$((y - 6) / 3) * (x + 4)$$

$$y - 6 = y\ 6\ -$$

$$((y\ 6\ -) / 3) * (x + 4)$$

The next operator can be looked at.

$$((y\ 6\ -)\ /\ 3)\ *\ (x + 4)$$

It is a divisor. The two pieces of operand surrounding it is 3 and the result of y 6 -.

$$((y\ 6\ -)\ /\ 3)\ *\ (x + 4)$$

This may seem confusing, but remember, y 6 - can be evaluated (with a value of y), so it can be treated as a single term.

$$((y\ 6\ -)\ /\ 3)\ *\ (x + 4)$$

$$(y\ 6\ -)\ /\ 3 = (y\ 6\ -)\ 3\ /$$

$$((y\ 6\ -)\ 3\ /)\ *\ (x + 4)$$

The next operator is observed.

$$((y\ 6\ -)\ 3\ /)\ *\ (x + 4)$$

The operand surrounding the multiplication sign is the result of the postfix expression ((y 6 -) 3 /) and the result of the infix expression (x + 4). Again, this is less complicated than it looks if each operand is taken as one term.

$$((y\ 6\ -)\ 3\ /)\ *\ (x + 4)$$

$$((y\ 6\ -)\ 3\ /)\ *\ (x + 4) =$$
$$((y\ 6\ -)\ 3\ /)\ (x + 4)\ *$$

$$((y\ 6\ -)\ 3\ /)\ (x + 4)\ *$$

It would be tempting to say that we have found the postfix equivalent - there isn't an operator to the right of the multiplication symbol. However, if we look back at the original equation, we can see that the + sign needs to be dealt with. Original equation:

$$((y - 6) / 3) * (x + 4)$$

Current equation:

$$((y\ 6\ -)\ 3\ /)\ (x + 4)\ *$$

The operand surrounding the plus sign is x and 4.

$$((y\ 6\ -)\ 3\ /)\ (x + 4)\ *$$

$$x + 4 = x\ 4\ +$$

$$((y\ 6\ -)\ 3\ /)\ (x\ 4\ +)\ *$$

Now all the opcode has been considered, the brackets can be removed as they are superfluous.

$$y\ 6\ -\ 3\ /\ x\ 4\ +\ *$$

Stacks

Stacks can be used to evaluate postfix equations. The algorithm goes along the array - operand is pushed onto the stack, whilst opcode causes two items to be popped off the stack with the result of the operation pushed onto the stack.

Synoptic Link

Stacks are a **data structure** with a **LIFO** (Last In, First Out) **order of execution**. Items **added** onto a stack are said to be "**pushed**". An item is **removed** by "**popping**".

Stacks are covered in **Stacks** under **Fundamentals of Data Structures**.

Algorithm

An algorithm is a set of instructions which completes a task in a finite time and always terminates.

Example 1:

The following RPN expression needs to be evaluated:

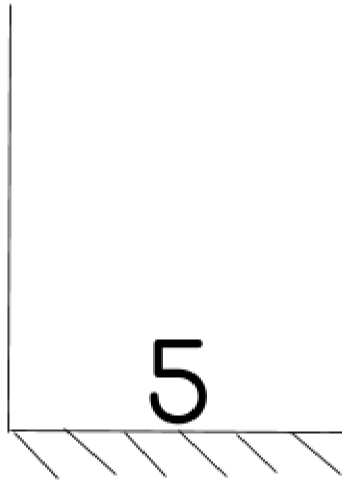
5 3 - 4 +

The leftmost item is selected first.

5 3 - 4 +

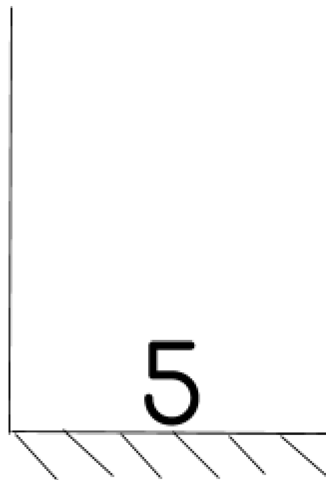
5 is the operand so it is pushed onto the stack.

5 3 - 4 +



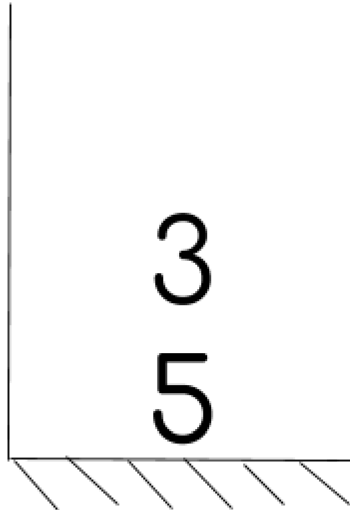
The next item is looked at.

5 3 - 4 +



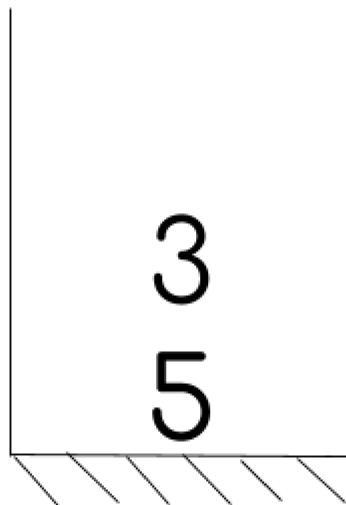
3 is also operand so it is pushed onto the stack.

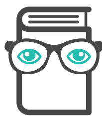
5 3 - 4 +



The next item is investigated.

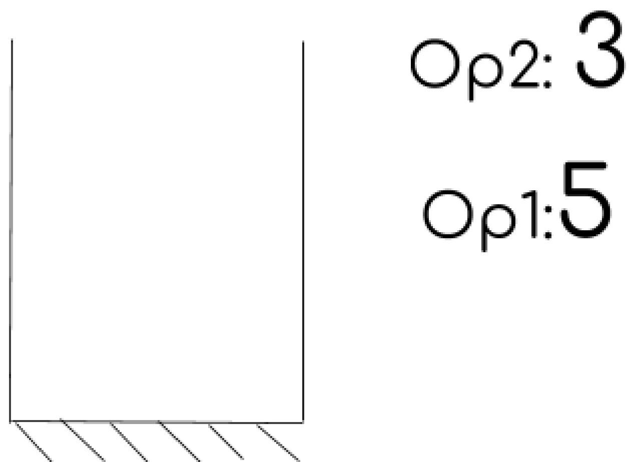
5 3 - 4 +





The minus sign is an operator. Therefore two items are popped off the stack - they will be the operand for this operation. First pop:

The 3 has been labelled as operand 2, this will help show the order of operation. Second pop:



Note

Remember the first item to be popped off the stack is the second operand (in infix, it is the operand which goes after the operator).

Now we have the opcode and the operand, an equation can be evaluated.

5 3 - 4 +

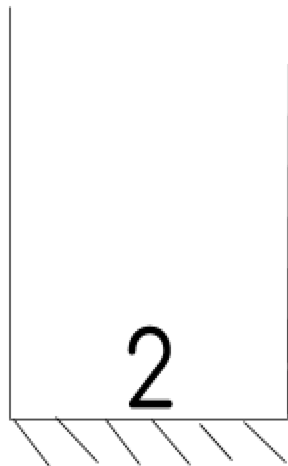
Op2: 3

Op1:5

5 - 3

The result is then pushed onto the stack.

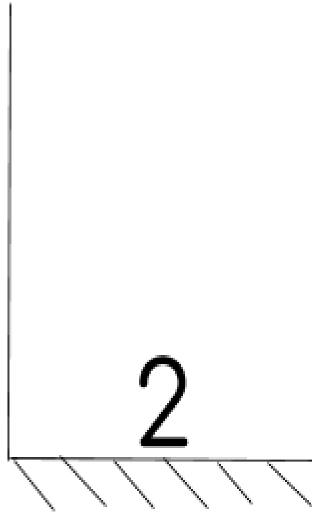
5 3 - 4 +



5 - 3 = 2

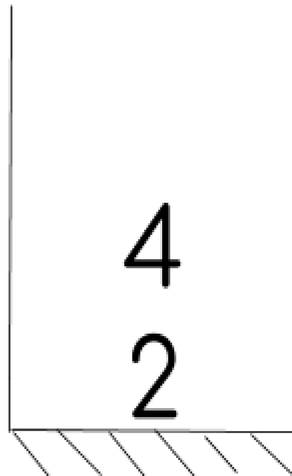
Now, the next item is looked at.

5 3 - 4 +



4 is the operand so it is pushed onto the stack.

5 3 - 4 +



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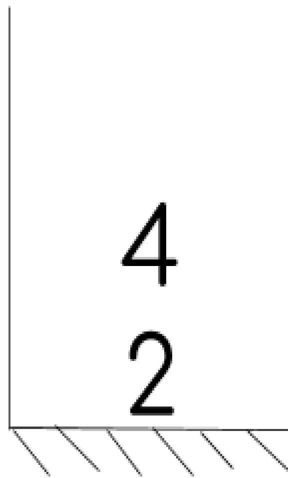
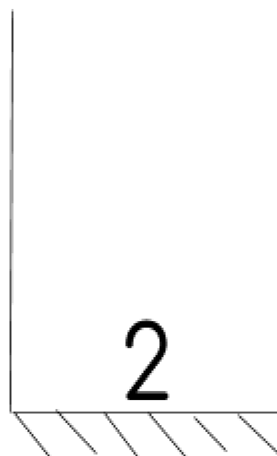
The next item is observed.

5 3 - 4 +



The addition sign is an operator, so two items are popped off the stack. First pop:

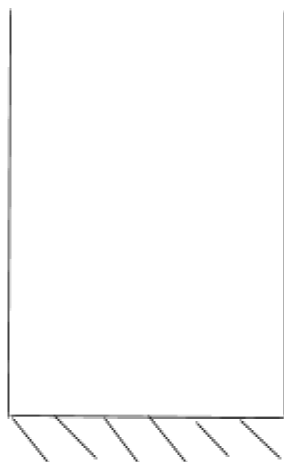
5 3 - 4 +



Op2: 4

Second pop:

5 3 - 4 +

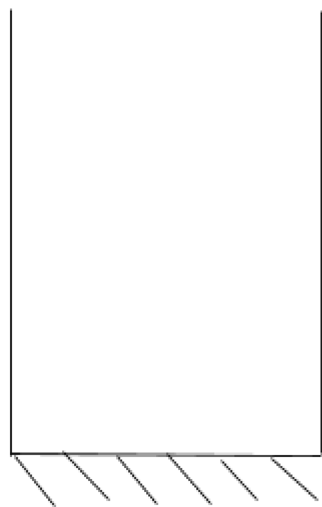


Op2: 4

Op1: 2

The operation can now be performed.

5 3 - 4 +



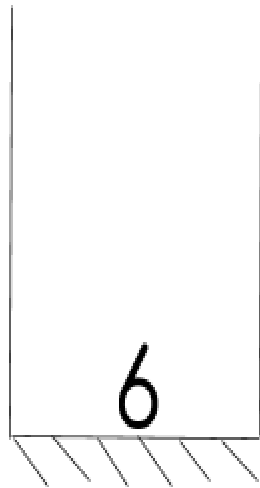
Op2: 4

Op1: 2

2 + 4

The answer is pushed onto the stack.

5 3 - 4 +



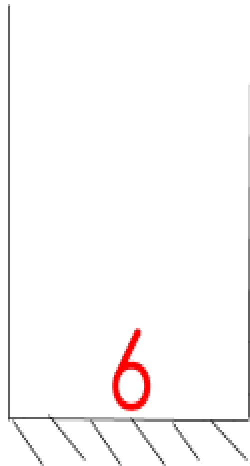
2 + 4 = 6

The next item is considered. There are no more items to consider.

5 3 - 4 +

The top of the stack is returned as the answer. The algorithm terminates.

5 3 - 4 + = 6



RPN Uses

As seen above, RPN can be executed on a stack. Due to this, RPN is ideal for interpreters which are based on a stack, e.g. Bytecode and PostScript. For more information, follow the links listed in the extra resources section.

> **Synoptic Link**
>
> **Interpreters** are a type of programming language translator which translates and executes instructions sequentially.
>
> Interpreters are covered in **Types of Program Translator** under **Fundamentals of Computer Systems.**